LECTURE: 3-6 DERIVATIVES OF LOGARITHMIC FUNCTIONS

Review: Derivatives of Exponential Functions: • $\frac{d}{dx}e^x =$ ______ • $\frac{d}{dx}a^x =$ ______

Example 1: Find a formula for the derivatives of the following functions.

(a) $y = \ln x$ (b) $y = \log_b x$

Derivatives of Logarithmic Functions: • $\frac{d}{dx} \ln x =$ _____ • $\frac{d}{dx} \log_b x =$ _____

Example 2: Find derivatives of the following functions.

(a) $y = \ln(4x^2 + 5)$ (b) $y = \ln(\tan x)$

Example 3: Find derivatives of the following functions.

(a)
$$f(x) = \log_{10} \sqrt{x}$$
 (b) $g(x) = \log_2(\cos x)$

Example 4: Differentiate f and find the domain of f'.

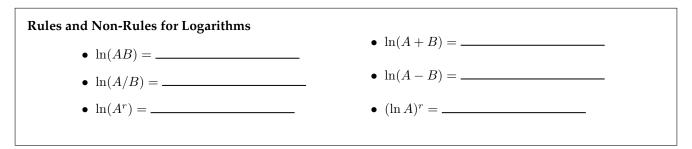
(a)
$$f(x) = \sqrt{5 + \ln x}$$
 (b) $f(x) = \frac{x}{1 - \ln(x + 1)}$

Example 5: Differentiate the following functions.

(a) $y = \ln |x|$.

(b) $f(x) = \ln |\sec x + \tan x|$

It is often easier to first use the rules of logarithms to expand a logarithmic expression before taking the derivative. To do this properly you first must recognize when these rules can be applied and apply them correctly.



Example 6: Differentiate the following functions by first expanding the expressions using the rules for logarithms. Explain *why* this is the better way to proceed in each case.

(a) $f(x) = \ln \sqrt{5x+2}$

(b)
$$g(x) = \log_5(x^2\sqrt{x+1})$$

Example 7: Differentiate $f(x) = \ln\left(\frac{x(x^2+1)^2}{\sqrt{2x^4-5}}\right)$

Example 8: Differentiate the following functions.

(a)
$$f(x) = (\ln x)^5$$
 (b) $f(x) = \ln x^5$

Logarithmic Differentiation

Finding derivatives of complicated functions involving products, quotients and powers can often be simplified using logarithms. This technique is called logarithmic differentiation.

Example 9: Find the derivative of $y = \frac{x^7\sqrt{x^3+1}}{(5x+1)^4}$.

Derivative Rules: Let *n* and *a* be constants. (Note, there is no rule when there is a variable in the base *and* the exponent.)

•
$$\frac{d}{dx}x^n =$$
 _____ • $\frac{d}{dx}a^x =$ _____

When you have a variable in both the base and the exponent you **must** use

_ to find the derivative of the function.

Example 10: Find the derivatives of the following functions using logarithmic differentiation.

(a) $y = x^{2/x}$

(b) $y = (\ln x)^{\cos x}$

Example 11: Find an equation of the tangent line to $f(x) = \ln(x + \ln x)$ at x = 1.

Example 12: Let $f(x) = cx + \ln(\sin x)$. For what value of c is $f'(\pi/4) = 6$?