LECTURE: 3-6 DERIVATIVES OF LOGARITHMIC FUNCTIONS

Review: Derivatives of Exponential Functions:

- $\frac{d}{dx}e^x =$ _____
- $\frac{d}{dx}a^x =$ _____

Example 1: Find a formula for the derivatives of the following functions.

(a) $y = \ln x$

(b) $y = \log_b x$

Derivatives of Logarithmic Functions:

- $\frac{d}{dx}\ln x =$ _____
- $\frac{d}{dx}\log_b x =$ _____

Example 2: Find derivatives of the following functions.

(a) $y = \ln(4x^2 + 5)$

(b) $y = \ln(\tan x)$

Example 3: Find derivatives of the following functions.

(a) $f(x) = \log_{10} \sqrt{x}$

(b) $g(x) = \log_2(\cos x)$

Example 4: Differentiate f and find the domain of f'.

(a) $f(x) = \sqrt{5 + \ln x}$

(b) $f(x) = \dfrac{x}{1 - \ln(x+1)}$

Example 5: Differentiate the following functions.

(a) $y = \ln|x|$.

(b) $f(x) = \ln|\sec x + \tan x|$

It is often easier to first use the rules of logarithms to expand a logarithmic expression before taking the derivative. To do this properly you first must recognize when these rules can be applied and apply them correctly.

Rules and Non-Rules for Logarithms

- $\ln(AB) =$ _____
- $\ln(A/B) =$ _____
- $\ln(A^r) =$ _____
- $\ln(A+B) =$ _____
- $\ln(A-B) =$ _____
- $(\ln A)^r =$ _____

Example 6: Differentiate the following functions by first expanding the expressions using the rules for logarithms. Explain *why* this is the better way to proceed in each case.

(a) $f(x) = \ln\sqrt{5x+2}$

(b) $g(x) = \log_5(x^2\sqrt{x+1})$

Example 7: Differentiate $f(x) = \ln\left(\dfrac{x(x^2+1)^2}{\sqrt{2x^4-5}}\right)$

Example 8: Differentiate the following functions.

(a) $f(x) = (\ln x)^5$

(b) $f(x) = \ln x^5$

Logarithmic Differentiation

Finding derivatives of complicated functions involving products, quotients and powers can often be simplified using logarithms. This technique is called logarithmic differentiation.

Example 9: Find the derivative of $y = \dfrac{x^7\sqrt{x^3+1}}{(5x+1)^4}$.

Derivative Rules: Let n and a be constants. (Note, there is no rule when there is a variable in the base *and* the exponent.)

- $\frac{d}{dx}x^n =$ _____
- $\frac{d}{dx}a^x =$ _____

When you have a variable in both the base and the exponent you **must** use

_____ to find the derivative of the function.

Example 10: Find the derivatives of the following functions using logarithmic differentiation.

(a) $y = x^{2/x}$

(b) $y = (\ln x)^{\cos x}$

Example 11: Find an equation of the tangent line to $f(x) = \ln(x + \ln x)$ at $x = 1$.

Example 12: Let $f(x) = cx + \ln(\sin x)$. For what value of c is $f'(\pi/4) = 6$?